An Introduction to Queuing Theory in an Interactive Text Format

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Abstract

This paper presents an interactive tutorial on queuing that uses Microsoft Excel as a platform. The unique features of this tutorial are related to a pedagogical plan that is appropriate for students of business. Implementation of the features in Excel is also discussed.

Introduction

Most people do not have an intuitive appreciation of random processes and the role that they play in managerial decisions. This is especially true of business students at the beginning stages of their curriculum. The queuing tutorial that is the subject of this paper was developed to introduce queuing models to an unsophisticated reader in such a way that (s)he could

- gain an intuitive understanding of the random processes that provide the underpinnings of queuing theory,
- learn how to use simple queuing models to support managerial decisions on resource allocation, and
- develop the ability to make critical judgments about the validity of the models.

Format

The format of this tutorial is a Microsoft Excel workbook (QingText.xls), in which sections are organized on different sheets. The written text material is placed on textboxes that float over the spreadsheet. Breaks occur in the textboxes at points where the reader is expected to interact with objects in the spreadsheet.

In the past decade, several authors (Bell, 2000; Plane, 1994; Powell, 1997; Savage, 1997) have discussed the value of live demonstrations and learning-by-doing made possible by using a computer spreadsheet in courses such as management science and statistics. To a large extent, manipulations with algebra and calculus and the algorithms of optimization and simulation are played down as students are taught to use common sense and their knowledge of business processes, along with spreadsheet formulas, functions and commands to create logical decision models. Along the way they also develop some valuable intuition about some mathematical concepts. We are beginning to see an emerging pedagogy of modeling (Powell, 2001) that helps to clarify and define the quantitative reasoning skills that are appropriate in a business program.

Today, there are many spreadsheet-oriented textbooks that are intended to be read at the computer, with the student actively participating in the creation of models as the textbook presents them. An added benefit is the ability of a spreadsheet to illuminate principles and concepts that are difficult to understand from straight text and figures. Savage (2001) calls these mindles (as in handles for the mind) and demonstrates how mindles can improve our intuitive understanding of random processes, sampling distributions, portfolio risk, etc.

As related by Tufte (1990), Galileo's observations of Saturn, published in 1613, interspersed icons within the set type to enhance the communication of the author's meaning. Extending this idea to electronic text, these icons might be animated "mindles" that help the reader to understand a statement. View Savage's (2001) paper online to see how animated graphical images can be used to convey meaning in an html document.

A printable version of this text in Adobe Acrobat format is also available.

Pedagogical Strategy

Anyone can plug numbers into queuing formulas and compute the results without gaining the ability to evaluate the validity of the model or to use the model for decision making. Beyond providing a survey of the simplest queuing models, this tutorial seeks to build an intuitive and conceptual understanding of the random processes involved. The reader then experiences the next logical step where the queuing performance results are used as inputs to a decision model. The focus is shifted from computations with queuing formulas to the creation of straightforward spreadsheet formulas to predict economic results.

After a brief background discussion, the tutorial opens with a simulation of aircraft landing at an airport. If you have QingText.xls open, click the "Index" sheet tab, then "Airport Simulation." There are several purposes to this demonstration.

The first is to give the reader a visual impression of a random process: sometimes aircraft arrive—other times they don't; sometimes there's no waiting line—other times it gets quite long. Calculating the average number of aircraft in the system (L = 3, in this case) creates the wrong impression—the reader imagines that one aircraft landing and two others circling overhead is the usual state of this system. The simulation creates a more realistic impression of a random process. Savage's "Flaw of Averages" Web page addresses this tendency of people to get erroneous perceptions from focusing on an average value rather than the complete probability distribution.

Another reason for the simulation is to get the reader to appreciate that the waiting lines are caused by the randomness of the arrival and service processes, even though the average capacity is sufficient to handle the average rate of arrivals.

Finally, the reader can adjust the arrival rate and service rate to see the effect on the waiting lines. But this will not be the immediate result that is seen by adjusting a parameter in a queuing formula—what the reader sees instead, after watching the simulation for several simulated hours, is a subtle change in the degree of congestion in the system.

The tutorial continues to build conceptual foundations by presenting a simulation of the Poisson process, a common assumption behind the random processes in queuing models (click "Poisson process" on the "Index" sheet in QingText.xls). This probability model is very easy to describe, but the simulation helps the reader to visualize how it is realized on a time scale.

The experiments with the graphs of exponential distributions ("Exponential distribution" on the "Index" sheet) are done for two reasons. One is to build awareness of the relationship between the mean of the exponential distribution (interarrival time or service time) and the Poisson parameter (arrival rate or service rate). This is a source of confusion for many students. The second is to make the student familiar with the shape of the density function and what it implies, because this assumption is often violated in applications of the simple queuing models. The reader needs a basis for making a judgment about whether service time would really fit this model in a real world application. The nature of the bias that is present in queuing model applications when the exponential distribution assumption is violated is discussed later in Example 5.

Since most of the computations for the M/M/1 model are fairly intuitive, those formulas are discussed next. Anything more complex requires the students to rely on software. The workbook Q.xls or the Queuing ToolPak add-in is used to compute waiting line characteristics for M/M/s, finite queue capacity, finite calling population, and M/G/1. The fact that either of these programs creates live, interactive results in spreadsheet cells means that it is a very natural next step to create formulas for economic evaluation. The student then has a cause-and-effect model in which the average cost of the system, for example, is linked to the number of servers. The reader can then optimize by trial and error. The more sophisticated students can readily create a data table and associated graph to illustrate the optimization, and to observe the sensitivity of the model to changes in assumptions about costs, service rates, etc. Examples 2, 3 and 4 develop these skills as they present illustrations of the different queuing models.

Excel Tips

Text boxes provide an easy way to incorporate text into a spreadsheet. Formatting possibilities are much more limited than what can be done with a word processor, but it is possible to mix fonts. Graphic objects and other text boxes can be floated on top of the text box, as necessary. For example, several text boxes are visible on the Intro worksheet. The first segment of the text starts at the top and extends to row 83, where an Excel demonstration begins.

Comments are very useful in an instructional spreadsheet. They are like text boxes that pop into view when the cursor passes over the cell to which they are assigned. They can be used for instructions, definitions, range names, and, of course, comments. They have been employed in the demonstration that begins at row 15 on the "Ex. 2" sheet. Passing the cursor over several of the cells, for example E16, shows the range name that has been applied to that cell. This makes it easier to understand the formulas in I20 and I22.

Visual Basic macros can be assigned to objects to make it easy for the reader to execute a demonstration. At row 85 on the "Intro" sheet is a simulation of a waiting line based on a Poisson arrival process and a Poisson service process. Clicking on the box that says, "Run the Clock," executes a macro that simulates the Poisson processes for as long as the reader specifies in cell G87.

The "Index" sheet is just a collection of text boxes, each with a macro assigned. Each macro selects a specific cell so that the reader views the appropriate text.

Data tables allow the user to experiment with changing functional forms and parameters, recalculating immediately to show the new results. This is particularly effective when used with charts. The example on the "Ex. 3" sheet starting in row 70 employs a chart showing the relationship between total cost and number of servers. The graph can be dragged to the vicinity of the parameter cells, allowing the reader to type in changes to the parameters and observe their effect on the cost relationship.

References

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